Published by IOP Publishing for SISSA

Received: *September 2, 2008*
Accepted: *October 13, 2008*
Published: *October 14, 2008*

Supergravity dual of Chern-Simons Yang-Mills theory with $\mathcal{N} = 6, 8$ superconformal IR fixed point

Akikazu Hashimoto and Peter Ouyang

Department of Physics, University of Wisconsin,
Madison, WI 53706, U.S.A.
E-mail: aki@physics.wisc.edu, pouyang@wisc.edu

Abstract: We construct a solution of eleven dimensional supergravity corresponding to a stack of M2 branes localized at the center of a particular eight dimensional hyper-Kähler manifold constructed by Gauntlett, Gibbons, Papadopoulos, and Townsend, generalizing the earlier construction of Cherkis and Hashimoto. In the decoupling limit, this solution is dual to a Chern-Simons/Yang-Mills/Matter theory in 2+1 dimensions with $\mathcal{N} = 3$ supersymmetry, which flows in the infra red to a superconformal Chern-Simons/Matter system preserving $\mathcal{N} = 6, 8$ supersymmetry, constructed recently by Aharony, Bergman, Jafferis, and Maldacena.

Keywords: Brane Dynamics in Gauge Theories, Gauge-gravity correspondence, AdS-CFT Correspondence, Chern-Simons Theories.

JHEP10(2008)057

Until recently, there was no known formulation of superconformal Chern-Simons theory in 2+1 dimensions with $\mathcal{N} = 8$ supersymmetry, and in fact the theory was believed not to exist [1]. This belief was reversed by the explicit construction of a model with $\mathcal{N} = 8$ supersymmetry by Bagger and Lambert [2, 3]. The original formulation of Bagger, Lambert, and Gustavsson involved the use of a 3-algebra, of which only a single finite dimensional example with a positive definite metric, the SO(4) 3-algebra, is known to exist [4, 5]. Shortly after its construction, this SO(4) model was shown to be equivalent to a more traditional Chern-Simons theory with an $\mathrm{SU}(2) \times \mathrm{SU}(2)$ gauge group, and matter fields in the bi-fundamental representation [6, 7] which does not rely on the use of a 3-algebra. These theories are extremely interesting as a candidate Lagrangian description of the decoupled field theory of M-theory membranes. In the past several months, there has been significant progress in the understanding of this model and its generalizations reported in the literature.

A very interesting new perspective on these class of models from the point of view of string theory was recently presented by Aharony et.al. [8]. These authors considered a configuration of branes in type IIB string theory involving D3-branes, NS5-branes, and (p, q) 5-branes of the form illustrated in figure 1. By (p, q) 5-brane, we mean the bound state of p NS5-branes and q D5-branes. More specifically, we orient the D3-branes along the 0126 directions. We take the 6 direction to be compact. The NS5-branes are oriented along the 012345 directions, and the (p, q) 5-branes are oriented along the $012[3, 7]_\theta[4, 8]_\theta[5, 9]_\theta$ directions. We are following the notational conventions of [8]. This brane configuration is a particular case of class of configurations considered in [9, 10] which generalizes the construction of Hanany-Witten type [11]. Localized intersection of (p, q) 5-brane and D3-brane was also studied recently in [12]. If $(p, q) = (1, 0)$, we recognize this system as describing an impurity system [13, 14] in 3+1 dimensions [15, 16] which flows to a 2+1 dimensional $\mathrm{U}(N)\times\mathrm{U}(N)$ Yang-Mills theory with bi-fundamental matter preserving $\mathcal{N} = 4$ supersymmetry. For $(p, q) = (1, k)$, one also obtains a defect field theory which flows to a $\mathrm{U}(N) \times \mathrm{U}(N)$ Yang-Mills theory with a Chern-Simons term at level k and matter in the bi-fundamental representations. These configurations generically preserve $\mathcal{N} = 3$ supersymmetry [9, 10]. The main observations of [8] are as follows:

- The level k $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons/Yang-Mills/matter theory flows in the IR to a level k $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons/matter theory with no Yang-Mills kinetic term.

- For $k > 2$, the IR theory has $\mathcal{N} = 6$ superconformal symmetry

- For $k = 1$ and $k = 2$, the supersymmetry of the IR theory is enhanced to $\mathcal{N} = 8$.

Aharony et.al. also noted that had they considered the gauge group $\mathrm{SU}(2) \times \mathrm{SU}(2)$, this model is equivalent to the product gauge group formulation [6, 7] of the Bagger-Lambert-Gustavsson theory. From this point of view, the role of the 3-algebra is demoted to the coincidence of the structure of the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ group, while the brane construction provides a plethora of models with $\mathcal{N} \geq 6$ supersymmetry where the features such as the

JHEP10(2008)057

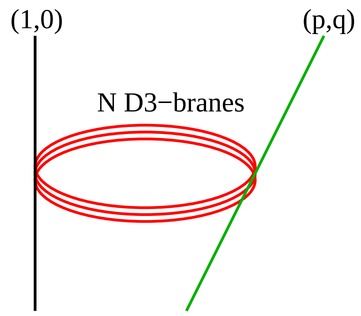

Figure 1: A configuration of D3, NS5, and (p, q) 5-branes in type IIB string theory. N D3-branes wind around an S_1 of size L. An NS5-brane and a (p, q) 5-brane intersects the D3-brane at a localized point along the S_1 but extends along the other 3 world volume coordinates of the D3-branes. Low energy effective theory of open strings is a Chern-Simons/Yang-Mills/matter theory with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$.

product gauge group and the bi-fundametal matter content have natural origins.[1]. The formulation of [8] also identifies the $\mathrm{U}(N) \times \mathrm{U}(N)$ at $k = 1$ as the candidate Lagrangian description for the stack of N M2-branes. Unfortunately, when $k = 1$ the model is strongly coupled, making the analysis of interesting features such as the $N^{3/2}$ scaling of the entropy beyond reach for the time being.

In order to see the enhancement of supersymmetry from $\mathcal{N} = 3$ to $\mathcal{N} = 6$ or 8 [8], it is useful to T-dualize the configuration of figure 1 along the 6 coordinate and lift to M-theory. This gives rise to a configuration of N M2-branes in eleven dimensions, compactified in 2 cycles, the (6,11), transverse to the world volume of the M2. The $(1, 0)$ and the (p, q) 5-branes are mapped to an overlapping configuration of KK5-branes with charged $(1, 0)$ and (p, q) with respect to the $\mathrm{U}(1) \times \mathrm{U}(1)$ associated with the 6 and 11 cycles, respectively. As it turns out, the complete supergravity description of these overlapping KK5-branes is known from the work of [18]. It can be described as an eight dimensional geometry with $sp(2)$ holonomy, and for general (p, q) gives a family of geometries generalizing Taub-NUT $\times$ Taub-NUT space which has holonomy group $sp(1) \times sp(1)$. With the $sp(2)$ holonomy, the geometry is hyper-Kähler and preserves 3/16 of the supersymmetries of the eleven dimensional supergravity, which is precisely what we expect for the dual of theories in 2+1 dimensions with $\mathcal{N} = 3$ supersymmetry. These spaces have also appeared as moduli-space of BPS monopoles [19, 20]. Just as in the case of the Taub-NUT geometry, the overlapping KK5-brane has a core region which is an orbifold C^4/Z_k where the discrete symmetry Z_k rotates each of the four complex plane in C^4 by an amount $2\pi/k$. Such an orbifold preserves 3/8 of the supersymmetry of eleven dimensional supergravity for $k > 2$ and 1/2 for $k = 1, 2$ [21]. Adding the M2 branes does not break any further supersymmetries. For a theory with large N and large 't Hooft coupling $\lambda = N/k$, we are lead to take the gravitational back reaction of the M2 branes into account, giving rise to a dual $AdS_4 \times S_7/Z_k$ geometry.

[1]A formulation of $\mathcal{N} = 6$ theory in terms of 3-algebra appeared in a recent article [17]

Let us now consider taking the limit where the cycle along the 6-direction, transverse to the M2-brane, is made arbitrarily large. This amounts to making the compact world volume of the D3-brane along the 6 direction in the original type IIB description, illustrated in figure 1, small. We would then have a Chern-Simons/Yang-Mills/matter system on the world volume of the D2-brane which we can decouple from gravity provided we scale the radius along the eleven direction appropriately.

It is possible to consider the supergravity dual of this configuration by taking the gravitational back reaction of the M2 branes into account. Such a description would be appropriate for large N. Finding the gravitational back reaction of the M2-brane amounts to finding a solution to Laplace's equation with a source in the background of the overlapping KK5-brane geometry. Once the Laplace equation is solved, it is straight forward to embed it into the solution to the equation of motion of eleven dimensional supergravity using the standard ansatz.

In fact, a problem very similar to this was discussed for the case where the KK5-brane geometry simplified to $R^4 \times$ Taub-NUT or Taub-NUT $\times$ Taub-NUT [22] where the holonomy group is $sp(1)$, and $sp(1) \times sp(1)$, respectively. The harmonic function is generically a solution to linear, partial differential equation. In [22], the Laplace equation was solved using brute force separation of variables. The resulting supergravity solution was interpretable as being dual to 2+1 dimensional SYM with matter in the fundamental representation. Regardless of the matter content, Yang-Mills theory in 2+1 dimensional is superrenormalizable, and as such, this supergravity solution is a dual of a UV complete field theory.

The goal of this paper is to solve for the analogous harmonic function for the overlapping KK5-brane geometry. By taking the appropriate decoupling limit, we obtain a supergravity solution which one can interpret as being dual to a specific Chern-Simons/Yang-Mills/matter theory in 2+1 dimensions. We will examine the form and the tractability of the Laplace equation in this background, with the expectation that the $sp(2)$ special holonomy should provide some degree of analytic control. Note that this precise program was outlined in the last paragraph of [22].

Let us begin by reviewing the basic ansatz for the intersecting brane configuration following [22]. We consider the ansatz

$$ds^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3} ds^2_{\mathcal{M}8} \tag{1}$$

$$F = dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} \tag{2}$$

where $\mathcal{M}_8$ is the eight dimensional $sp(2)$ holonomy manifold, and $H(y_i)$ is a scalar function depending only on the coordinates of $\mathcal{M}_8$. By substituting this ansatz into the equation of motion of supergravity in eleven dimensions, one can show that H is required to solve the Laplace equation in $\mathcal{M}_8$.

Next, let us review the metric for $\mathcal{M}_8$ [18]. It is given by

$$ds^2 = V_{ij} d\vec{y}_i d\vec{y}_j + (V^{-1})^{ij} R_i R_j (d\varphi_i + A_i)(d\varphi_j + A_j) \tag{3}$$

where

$$V_{ij} = \delta_{ij} + \frac{1}{2}\frac{R_i p_i R_j p_j}{|R_1 p_1 \vec{y}_1 + R_2 p_2 \vec{y}_2|} + \frac{1}{2}\frac{R_i \tilde{p}_i R_j \tilde{p}_j}{|R_1 \tilde{p}_1 \vec{y}_1 + R_2 \tilde{p}_2 \vec{y}_2|}\,, \tag{4}$$

i, j take values $1, 2$, and $\vec{y}_i$ are 3 vectors. We have restricted our attention to the case where there are two overlapping KK5-branes whose charges are

$$(p_1, p_2) = (1, 0), \qquad (\tilde{p}_1, \tilde{p}_2) = (1, k) \tag{5}$$

to match the construction of [8]. The φ_i coordinate is chosen to have period 2π. So R_1 and R_2 are radius of $S_1 \times S_1$ which we identify as the 6 and 11 directions, respectively. Therefore, when taking the decoupling limit, we scale

$$R_1 = \frac{2\pi\alpha'}{L}, \qquad R_2 = g_s l_s = c g_{YM2}^2 \alpha' \tag{6}$$

where $c = (2\pi)^{p-2} = 1$ for $p = 2$ [23], and L is the size of the circle along the 6-direction in the dual type IIB description illustrated in figure 1. In the decoupling limit, we must keep g_{YM2}^2 fixed, but we are free to vary L, and eventually we take $L \to 0$ to simplify the analysis. This limit corresponds to $R_2/R_1 \propto g_{YM2}^2 L \to 0$.

The simplest and the most symmetric case to consider is to place the M2-brane at the origin $\vec{y}_1 = \vec{y}_2 = 0$. We also restrict our attention to a solution symmetric with respect to shifts in φ_1 and φ_2. In the near core region, this is simply the rotational symmetry of the ansatz.

It is then straight forward to write the Laplace equation on this geometry as

$$0 = \partial_\mu(\sqrt{g} g^{\mu\nu} \partial_\nu H) = \vec{\partial}_i \det V (V^{-1})^{ij} \vec{\partial}_j H(\vec{y}_1, \vec{y}_2) \ . \tag{7}$$

This can be simplified a little by changing variables

$$\vec{r}_1 = \vec{y}_1, \qquad \vec{r}_2 = \vec{y}_2 + \frac{R_1}{kR_2} \vec{y}_1 \ . \tag{8}$$

The Laplace equation will then have the form

$$\left[\left(1 + \frac{kR_2}{2r_2}\right) \vec{\partial}_1^2 + \frac{2R_1}{kR_2} \vec{\partial}_1 \cdot \vec{\partial}_2 + \left(1 + \frac{R_1^2}{k^2 R_2^2} + \frac{R_1}{2r_1}\right) \vec{\partial}_2^2\right] H(\vec{r}_1, \vec{r}_2) \ . \tag{9}$$

The most symmetric configuration can depend, in general, on

$$r_1 = |\vec{r}_1|, \qquad r_2 = |\vec{r}_2|, \qquad z = \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2} \ . \tag{10}$$

In terms of these variables, the differential operators appearing in 9 have the form

$$\vec{\partial}_i^2 = \frac{1}{r_i} \left(\frac{\partial}{\partial r_i}\right)^2 r_i + \frac{1}{r_i^2} \left((1 - z^2)\partial_z^2 - 2z\partial_z\right) \tag{11}$$

$$\vec{\partial}_1 \cdot \vec{\partial}_2 = \frac{1}{r_1 r_2} \left(z(z^2 - 1)\partial_z^2 + (1 + z^2)\partial_z\right) + \frac{(1 - z^2)}{r_1} \partial_{r_2} \partial_z + \frac{(1 - z^2)}{r_2} \partial_{r_1} \partial_z + z \partial_{r_1} \partial_{r_2} \tag{12}$$

At this point, we are faced with a linear yet seemingly unseparable partial differential equation of three variables, with no obvious hope for any simplification.

We are, however, entitled to take the large R_1 limit. To do this, it is convenient to make the change of variables standard in taking the near core limit of a Taub-NUT geometry

$$r_1 = \frac{\rho_1^2}{2R_1}, \qquad r_2 = \frac{\rho_2^2}{2kR_2} \ . \tag{13}$$

In these coordinates, the metric on $\mathcal{M}_8$ has the form

$$\begin{aligned}
ds^2_{\mathcal{M}8} = &\left(1+\left(\frac{1}{R_1^2}+\frac{1}{k^2R_2^2}\right)\rho_1^2\right)\left(d\rho_1^2+\frac{\rho_1^2}{4}(d\theta_1^2+\sin^2\theta_1 d\phi_1^2)\right)\\
&+\left(1+\frac{\rho_2^2}{k^2R_2^2}\right)\left(d\rho_2^2+\frac{\rho_2^2}{4}(d\theta_2^2+\sin^2\theta_2 d\phi_2^2)\right)-\frac{\rho_1^2\rho_2^2}{2k^2R_2^2}\left(\frac{d\vec{r}_1\cdot d\vec{r}_2}{r_1r_2}\right)\\
&+\begin{pmatrix} \frac{1}{R_1^2}+\frac{1}{\rho_1^2}+\frac{1}{\rho_2^2} & \frac{k}{\rho_2^2} \\ \frac{k}{\rho_2^2} & \frac{1}{R_2^2}+\frac{k^2}{\rho_2^2}\end{pmatrix}^{-1\,ij}(d\varphi_i+A_i)(d\varphi_j+A_j)
\end{aligned} \tag{14}$$

where θ_i and ϕ_i are the angular coordinates in S^2 of $\vec{r}$. We have not reparameterized the term proportional to $d\vec{r}_1\cdot d\vec{r}_2/r_1r_2$ but it should be clear that this expression is independent of R_1. After taking $R_1\to\infty$ keeping ρ_i and R_2 fixed, the harmonic equation becomes

$$\begin{aligned}
0 = \Bigg[&\left(1+\frac{\rho_2^2}{k^2R_2^2}\right)\left(\partial_{\rho_1}^2+\frac{3}{\rho_1}\partial_{\rho_1}+\frac{4}{\rho_1^2}\left((1-z^2)\partial_z^2-2z\partial_z\right)\right)\\
&+\left(1+\frac{\rho_1^2}{k^2R_2^2}\right)\left(\partial_{\rho_2}^2+\frac{3}{\rho_2}\partial_{\rho_2}+\frac{4}{\rho_2^2}\left((1-z^2)\partial_z^2-2z\partial_z\right)\right)\\
&+\frac{2\rho_1^2\rho_2^2}{k^2R_2^2}\left(\frac{\vec{\partial}_1\cdot\vec{\partial}_2}{kR_1R_2}\right)\Bigg]H(\rho_1,\rho_2,z)
\end{aligned} \tag{15}$$

where the expression

$$\begin{aligned}
\frac{\vec{\partial}_1\cdot\vec{\partial}_2}{kR_1R_2} = &\frac{4}{\rho_1^2\rho_2^2}\left(z(z^2-1)\partial_z^2+(1+z^2)\partial_z\right)\\
&+\frac{2}{\rho_1^2\rho_2}(1-z^2)\partial_{\rho_2}\partial_z+\frac{2}{\rho_1\rho_2^2}(1-z^2)\partial_{\rho_1}\partial_z+\frac{z}{\rho_1\rho_2}\partial_{\rho_1}\partial_{\rho_2}
\end{aligned} \tag{16}$$

is independent of R_1 despite appearances. Although this equation is still not separable, we see that if $R_2\to\infty$, this equation simplifies to

$$\begin{aligned}
0 = \Bigg[&\left(\partial_{\rho_1}^2+\frac{3}{\rho_1}\partial_{\rho_1}+\frac{4}{\rho_1^2}\left((1-z^2)\partial_z^2-2z\partial_z\right)\right)\\
&+\left(\partial_{\rho_2}^2+\frac{3}{\rho_2}\partial_{\rho_2}+\frac{4}{\rho_2^2}\left((1-z^2)\partial_z^2-2z\partial_z\right)\right)\Bigg]H(\rho_1,\rho_2,z)
\end{aligned} \tag{17}$$

which is separable. An obvious solution is

$$H_0 = \frac{c' N l_p^6}{(\rho_1^2+\rho_2^2)^3}\,, \tag{18}$$

where $c' = 2^5\pi^2$ [24]. Such simplicity is exactly what we expect because when $R_2\to\infty$ we are working in the near core limit where $\mathcal{M}_8 = C^4/Z_k$.

JHEP10(2008)057

Let us now look at how the harmonic equation depends on R_2. One can in fact collect its dependence on R_2 and write down a recursion relation

$$\mathbf{A}H_{i+1}(\rho_1, \rho_2, z) = -\mathbf{B}H_i(\rho_1, \rho_2, z) \tag{19}$$

where $\mathbf{A}$ and $\mathbf{B}$ are differential operators

$$\begin{aligned}
\mathbf{A} &= \left(\partial_{\rho_1}^2 + \frac{3}{\rho_1}\partial_{\rho_1} + \frac{4}{\rho_1^2}\left((1-z^2)\partial_z^2 - 2z\partial_z\right)\right) + \left(\partial_{\rho_2}^2 + \frac{3}{\rho_2}\partial_{\rho_2} + \frac{4}{\rho_2^2}\left((1-z^2)\partial_z^2 - 2z\partial_z\right)\right) \\
\mathbf{B} &= \left[\frac{\rho_2^2}{k^2R_2^2}\left(\partial_{\rho_1}^2 + \frac{3}{\rho_1}\partial_{\rho_1} + \frac{4}{\rho_1^2}\left((1-z^2)\partial_z^2 - 2z\partial_z\right)\right)\right. \\
&\quad \left. + \frac{\rho_1^2}{k^2R_2^2}\left(\partial_{\rho_2}^2 + \frac{3}{\rho_2}\partial_{\rho_2} + \frac{4}{\rho_2^2}\left((1-z^2)\partial_z^2 - 2z\partial_z\right)\right) + \frac{2\rho_1^2\rho_2^2}{k^2R_2^2}\left(\frac{\vec{\partial}_1 \cdot \vec{\partial}_2}{kR_1R_2}\right)\right] .
\end{aligned} \tag{20}$$

This means

$$H_i = (-\mathbf{A}^{-1}\mathbf{B})^i H_0 \tag{21}$$

and

$$H = \sum_i H_i = \frac{1}{1 + \mathbf{A}^{-1}\mathbf{B}} H_0 . \tag{22}$$

Although this expression for the solution appears rather formal, it is acceptable if the operator $\mathbf{A}$ is invertible, or in other words that the zero-modes of $\mathbf{A}$ may be projected out by appropriate boundary conditions for large ρ_i. In fact, after separating variables, one obtains the differential equation

$$\left((1-z^2)\partial_z^2 - 2z\partial_z + n(n+1)\right) f(z) = 0 \tag{23}$$

which is solved by

$$f(z) = L_n(z) \tag{24}$$

where $L_n(z)$ is the Legendre polynomial of degree n. This is the natural basis to work in when acting with $\mathbf{A}^{-1}$. To generate the recursive sum, one must act with $\mathbf{B}$, expand the z dependence in Legendre polynomial basis, and convolve the Green's function with respect to ρ_1 and ρ_2. As a proof of principle, we will compute the first few terms in the expansion. Physically, this computation captures the behavior of the RG flow close to the superconformal IR fixed point.

An effective technique for computing the action of $\mathbf{A}^{-1}$ is the method of undetermined coefficients. Acting with $\mathbf{B}$, we find that

$$\mathbf{B}H_0 = \left(\frac{96z\rho_1^2\rho_2^2}{k^2R_2^2(\rho_1^2+\rho_2^2)^5} - \frac{24(\rho_1^2-\rho_2^2)^2}{k^2R_2^2(\rho_1^2+\rho_2^2)^5}\right) c'Nl_p^6. \tag{25}$$

We then consider a general linear combination of basis functions for which action by $\mathbf{A}$ produces terms of the form in (25). The basis functions must satisfy the following properties. First, they may only depend on ρ_i through ρ_i^2, and must be symmetric under interchanging ρ_1 and ρ_2. Second, for physical reasons we expect poles only of the form $\frac{1}{(\rho_1^2+\rho_2^2)^n}$. Third, H_1 should not contain any factors more divergent than $\frac{1}{(\rho_1^2+\rho_2^2)^4}$. Once the power of $\frac{1}{(\rho_1^2+\rho_2^2)}$

is established, there will be an additional coefficient of $\rho_1^2\rho_2^2$ as determined by dimensional analysis (up to a change of basis functions.) This motivates the ansatz

$$k^2 R_2^2 H_1 = c_1 \frac{z\rho_1^2\rho_2^2}{(\rho_1^2+\rho_2^2)^4} + c_2 \frac{z}{(\rho_1^2+\rho_2^2)^2} + c_3 \frac{\rho_1^2\rho_2^2}{(\rho_1^2+\rho_2^2)^4} + c_4 \frac{1}{(\rho_1^2+\rho_2^2)^2} \tag{26}$$

which solves the first stage of the recursion relation for $c_1 = 2, c_2 = 0, c_3 = 2, c_4 = -1$, or

$$H_1 = \frac{c' N l_p^6}{k^2 R_2^2} \left(\frac{2(1+z)\rho_1^2\rho_2^2}{(\rho_1^2+\rho_2^2)^4} - \frac{1}{(\rho_1^2+\rho_2^2)^2} \right) . \tag{27}$$

A similar calculation produces for H_2:

$$H_2 = \frac{c' N l_p^6}{k^4 R_2^4} \left(\frac{8}{3} \frac{(1+z)^2 \rho_1^4\rho_2^4}{(\rho_1^2+\rho_2^2)^5} - \frac{26}{9} \frac{(1+z)\rho_1^2\rho_2^2}{(\rho_1^2+\rho_2^2)^3} + \frac{10}{9} \frac{1}{(\rho_1^2+\rho_2^2)} \right) . \tag{28}$$

At each order in the recursion, there are finitely many basis functions, so this method can be applied at any order. It quickly becomes clear, though, that at higher orders the explicit calculations become quite cumbersome. Moreover, a problem of a different sort begins to appear at the third order in the recursion — equation (19) is satisfied for $AH_3 = -BH_2$ where we may add any constant to H_3. In a complete solution, this zero-mode would be fixed by the behavior of $H(\rho_1, \rho_2)$ at large ρ_i, after the resummation, and our perturbative solution is inadequate for this purpose. Nevertheless, H_1 and H_2 do appear to have some pattern suggesting that perhaps there is some way to resum this series, or that perhaps the harmonic function can be computed numerically.

By substituting this solution into the ansatz (2), and scaling, as usual for the M2-branes [24],

$$\rho_1 = l_p^{3/2} U_1^{1/2}, \qquad \rho_2 = l_p^{3/2} U_2^{1/2}, \tag{29}$$

we will obtain a supergravity dual of the decoupled field theory. To see the structure of this solution, let us first examine the scaling of the metric of $\mathcal{M}_8$

$$ds^2_{\mathcal{M}8} = l_p^3 dS^2_{\mathcal{M}8}(U_1, \theta_1, \phi_1, \varphi_1, U_2, \theta_2, \phi_2, \varphi_2) \tag{30}$$

where

$$\begin{aligned} dS^2_{\mathcal{M}8} = & \left(1 + \frac{U_1}{ck^2 g^2_{YM2}}\right)\left(\frac{1}{4U_1} dU_1^2 + \frac{U_1}{4}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2)\right) \\ & + \left(1 + \frac{U_2}{k^2 c g^2_{YM2}}\right)\left(\frac{1}{4U_2} dU_2^2 + \frac{U_2}{4}(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2)\right) \\ & - \frac{U_1 U_2}{2cg^2_{YM2} k^2}\left(\frac{d\vec{r}_1 \cdot d\vec{r}_2}{r_1 r_2}\right) + \begin{pmatrix} \frac{1}{U_1} + \frac{1}{U_2} & \frac{k}{U_2} \\ \frac{k}{U_2} & \frac{1}{cg^2_{YM2}} + \frac{k^2}{U_2} \end{pmatrix}^{-1\,ij} (d\varphi_i + A_i)(d\varphi_j + A_j) \end{aligned} \tag{31}$$

has the dimension of inverse length and is independent of l_p. We have expressed R_2 in terms of the field theory parameter

$$R_2^2 = c g^2_{YM2} l_p^3 \tag{32}$$

by combining (6) with the standard relation

$$l_p = g^{1/3} l_s \ . \tag{33}$$

Let us also introduce a scaled harmonic function

$$h(U_1, \theta_1, \phi_1, U_2, \theta_2, \phi_2) = l_p^3 H \tag{34}$$

which is also independent of l_p. Using (18) as H_0, we have

$$\begin{aligned}
h_0 &= l_p^3 H_0 = \frac{c' N}{(U_1 + U_2)^3} \\
h_1 &= l_p^3 H_1 = \frac{c' N}{c g_{YM2}^2 k^2} \left(\frac{2(1+z) U_1 U_2}{(U_1 + U^2)^4} - \frac{1}{(U_1 + U_2)^2} \right) \\
h_2 &= l_p^3 H_2 = \frac{c' N}{c^2 g_{YM}^4 k^4} \left(\frac{8(1+z)^2 U_1^2 U_2^2}{3(U_1 + U_2)^5} - \frac{26(1+z) U_1 U_2}{9(U_1 + U_2)^3} + \frac{10}{9(U_1 + U_2)} \right)
\end{aligned} \tag{35}$$

which indeed is independent of l_p. In terms of these quantities, the supergravity solution we are after takes the form

$$ds^2 = l_p^2 \left[h^{-2/3} (-dt^2 + dx_1^2 + dx_2^2) + h^{1/3} dS_{\mathcal{M}8} \right] \tag{36}$$

where the expression inside the square bracket only depends on field theory variables and not on l_p. Also, for small U_1 and U_2, the geometry asymptotes to $AdS_4 \times S_7/Z_k$. It is also straight forward to reduce this geometry to type IIA. These geometries capture the renormalization group flow of Chern-Simons/Yang-Mills/matter system down to $\mathcal{N} = 6, 8$ superconformal Chern-Simons/matter theory, and is effective for $N \gg k$.

The explicit solution to the eleven dimensional supergravity equations of motion given in (22), (31), (35), and (36) is the main result of this paper. Admittedly, the solution we found is not in an ideal form. The recursive nature of the solution presented here makes it cumbersome to evaluate and display the function even using numerical methods. Still, the form that the solution takes for large and small U_i was clear from the beginning. The recursive procedure provides the details of the solution near the cross-over region at the scale $g_{YM2}^2 k$ corresponding to the mass deformation due to the Chern-Simons term.

The eight dimensional hyper-Kähler geometry we studied in this paper has quite a bit of structure [18]. The fact that a recursive procedure for solving for the Greens function on this space suggests the possibility that there exists more elegant approaches to the problem we considered. Green's functions in self-dual four manifolds have been analyzed using various methods [25, 26]. Perhaps some of these methods can be applied to the problem considered in this paper.

Acknowledgments

We would like to thank O. Aharony, O. Bergman, S. Cherkis, S. Hirano, N. Itzhaki, and J. Sonnenschein for discussions. AH is grateful to Tel Aviv University and Weizmann Institute where part of this work was done. This work was supported in part by the DOE grant DE-FG02-95ER40896, NSF CAREER Award No. PHY-0348093, a Cottrell Scholar Award from Research Corporation, and funds from the University of Wisconsin.

JHEP10(2008)057

References

[1] J.H. Schwarz, *Superconformal Chern-Simons theories*, *JHEP* **11** (2004) 078 [hep-th/0411077].

[2] J. Bagger and N. Lambert, *Gauge symmetry and supersymmetry of multiple* $M2$*-branes*, *Phys. Rev.* **D 77** (2008) 065008 [arXiv:0711.0955].

[3] A. Gustavsson, *Algebraic structures on parallel* $M2$*-branes*, arXiv:0709.1260.

[4] G. Papadopoulos, $M2$*-branes,* 3*-Lie algebras and Plücker relations*, *JHEP* **05** (2008) 054 [arXiv:0804.2662].

[5] J.P. Gauntlett and J.B. Gutowski, *Constraining maximally supersymmetric membrane actions*, arXiv:0804.3078.

[6] M. Van Raamsdonk, *Comments on the Bagger-Lambert theory and multiple* $M2$*-branes*, *JHEP* **05** (2008) 105 [arXiv:0803.3803].

[7] M.A. Bandres, A.E. Lipstein and J.H. Schwarz, $N = 8$ *superconformal Chern-Simons theories*, *JHEP* **05** (2008) 025 [arXiv:0803.3242].

[8] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, $N = 6$ *superconformal Chern-Simons-matter theories,* $M2$*-branes and their gravity duals*, arXiv:0806.1218.

[9] T. Kitao, K. Ohta and N. Ohta, *Three-dimensional gauge dynamics from brane configurations with* (p, q)*-fivebrane*, *Nucl. Phys.* **B 539** (1999) 79 [hep-th/9808111].

[10] O. Bergman, A. Hanany, A. Karch and B. Kol, *Branes and supersymmetry breaking in* 3*D gauge theories*, *JHEP* **10** (1999) 036 [hep-th/9908075].

[11] A. Hanany and E. Witten, *Type IIB superstrings, BPS monopoles and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

[12] D. Gaiotto and E. Witten, *Janus configurations, Chern-Simons couplings, and the* θ*-angle in* $N = 4$ *super Yang-Mills theory*, arXiv:0804.2907.

[13] S. Sethi, *The matrix formulation of type IIB five-branes*, *Nucl. Phys.* **B 523** (1998) 158 [hep-th/9710005].

[14] A. Kapustin and S. Sethi, *The Higgs branch of impurity theories*, *Adv. Theor. Math. Phys.* **2** (1998) 571 [hep-th/9804027].

[15] A. Karch and L. Randall, *Localized gravity in string theory*, *Phys. Rev. Lett.* **87** (2001) 061601 [hep-th/0105108].

[16] O. DeWolfe, D.Z. Freedman and H. Ooguri, *Holography and defect conformal field theories*, *Phys. Rev.* **D 66** (2002) 025009 [hep-th/0111135].

[17] J. Bagger and N. Lambert, *Three-algebras and* $N = 6$ *Chern-Simons gauge theories*, arXiv:0807.0163.

[18] J.P. Gauntlett, G.W. Gibbons, G. Papadopoulos and P.K. Townsend, *Hyper-Kähler manifolds and multiply intersecting branes*, *Nucl. Phys.* **B 500** (1997) 133 [hep-th/9702202].

[19] G.W. Gibbons and N.S. Manton, *The moduli space metric for well separated BPS monopoles*, *Phys. Lett.* **B 356** (1995) 32 [hep-th/9506052].

[20] K.-M. Lee, E.J. Weinberg and P. Yi, *The moduli space of many BPS monopoles for arbitrary gauge groups*, *Phys. Rev.* **D 54** (1996) 1633 [hep-th/9602167].

JHEP10(2008)057

[21] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, *Class. and Quant. Grav.* **1** (1984) 499.

[22] S.A. Cherkis and A. Hashimoto, *Supergravity solution of intersecting branes and AdS/CFT with flavor*, *JHEP* **11** (2002) 036 [hep-th/0210105].

[23] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large-N limit of theories with sixteen supercharges*, *Phys. Rev.* **D 58** (1998) 046004 [hep-th/9802042].

[24] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[25] D.N. Page, *Green's functions for gravitational multi-instantons*, *Phys. Lett.* **B 85** (1979) 369.

[26] M.F. Atiyah, *Green's functions for selfdual four manifolds*, *Adv. Math. Supp.* **7A** (1981) 129.